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There has recently been an interest in graduated payment mortgages as a technique to allow farmland to amortize its own debt with the annual cash flow it generates itself. Graduated payments are normally only effective, however, when the first payment is lower than the first period's accrued interest. This results in an increase in the unpaid balance of the loan, a result that neither the lender nor the borrower relish.

This article looks at the use of simple interest with graduated payment schedules. Although simple interest would normally not be used by a commercial lender, it is an attractive option to a farm seller who will report taxable gain on an installment basis, and is required to charge at least 9 percent simple interest. Simple interest graduated payments might also be used between family members as a viable technique to transfer the farm by sale to the next generation. The attraction of simple interest is that no matter how small the first payment, the same proportion of that first payment will be used for principal reduction. Thus, there will never be an increase in the unpaid balance of the loan.

The Necessity for Graduated Payments

In recent years farmland has displayed an inability to liquidate its own debt since the cash flow generated by the farmland is insufficient initially to amortize a conventional loan. It has been necessary for farmers to supplement the cash generated from the farmland with other farm or non-farm income. This has inhibited potential buyers from acquiring farmland because they lack other sources of income, even if they could be the most productive users of the farmland.

One reason farmland is unable to liquidate its own debt is because it has been characterized as a growth stock (Melichar). If the earnings from farmland are expected to increase into the future, then the farmland will sell for a high multiple of its current earnings, with those current earnings being insufficient initially to pay for the farmland with conventional debt payments.

An example of this phenonemon can be observed by looking at twenty years of change in farm investment and cash income on New York dairy farms, Table 1. From 1963 to 1982 cash income per cow increased at an average annual rate of 5 percent, although the increase was not uniform and decreases did occur. At the same time investment per cow increased at an annual rate of 7 percent with increases occurring each year except 1982. Even more significant, however, is that investment per dollar of cash income increased from about 7 during the first ten years to about 12 for the last ten years. Ignoring living and capital replacement requirements, it is apparent that it has now become more difficult to pay for a dairy farm from its earnings.

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Table 1. Twenty Years of Change in Dairy Farm Investment and Cash Income
Dairy Farm Business Management Cooperators, New York, 1963-1982

Year	Cash Income Per Cow	Farm Investment Per Cow	Investment / Cash Income	Number of Cows
1982	$ 440	$ 5,516	12.54	82
81	470	5,820	12.38	79
80	479	5,686	11.87	75
79	473	5,266	11.13	75
78	382	4,540	11.88	71
77	332	4,003	12.06	71
76	361	3,706	10.27	71
75	276	3,447	12.49	72
74	292	3,216	11.01	72
73	261	3,009	11.53	69
72	312	2,482	7.96	70
71	319	2,288	7.17	67
70	307	2,112	6.88	65
69	305	2,020	6.62	60
68	290	1,930	6.66	58
67	260	1,800	6.92	51
66	246	1,714	6.97	47
65	227	1,565	6.89	44
64	183	1,466	8.01	40
63	176	1,450	8.24	39

Cash income is cash receipts minus cash expenses. (No depreciation nor capital expenditures included.) Farm investment includes livestock, feed and supplies, machinery and equipment, land and buildings.

With an investment of $5,500 per cow in 1982, no down payment, and 9 percent compound interest, it would take annual payments of $602 to pay for the farm in 20 years. This amount is significantly greater than the $440 that was available in 1982 before capital replacement and living needs. This can be contrasted with 1972 when, with an investment of $2,500 per cow, no down payment, and interest at 9 percent compounded, it would only have taken annual payments of $274 to pay for the farm in 20 years; $312 was available that first year.

A partial solution to the cash-flow shortage is to lengthen the repayment period. At an investment of $5,500, lengthening the payment period from 20 to 40 years at 9 percent compound interest lowers the annual payment from $602 to $511. This is still more than the $440 available.

A suggested remedy is to structure payments so that they start at a lower amount but increase over time, as income has. A number of researchers have demonstrated the potential of these graduated payment plans (Lee, Lins and Aukes). One encumbrance to these payments is that they require increasing income over time. Ellinger, Barry and Lins have shown that if annual earnings follow historical patterns, graduated payment mortgages could allow previously excluded farmers into the land market, and allow all farmers to expand faster. These graduated payment plans are all based upon geometric increases in payments with compound interest.

The paradox of these plans using compound interest is that, if the first year's payment is not an amount of at least the accrued interest, the unpaid balance of the loan will increase, requiring much larger payments later and encouraging default if the value of the farmland does not increase accordingly. Yet, in order to be useful, the first payments must be less than the accrued interest. For instance, the first year's accrued interest of 9 percent on $5,500 is $495. A 40 year level payment loan is only $16 more than this amount.

The concept of simple interest eliminates this difficulty. By definition, simple interest is paid on principal at the same time that the principal is paid. And then, the interest is simply the interest rate, times the principal paid, times the number of periods since the money was borrowed. As an example, assume $10,000 is borrowed for 3 years at 9 percent simple interest. When the $10,000 is repaid, then 3 x .09 x $10,000 or $2,700 interest is also paid. If interest were compounded annually, the interest payment at the end of 3 years would be $2,950. Such a difference in compound and simple interest also applies to any multiple payment plans, for they are simply strings of single payment loans (Tauer).

Simple interest would not normally be used by a commerical lender unless the interest rate was sufficiently high enough to equal the return from compounding. Simple interest is a viable option for seller financing however, especially when the seller is reporting taxable gain in installments which qualify for capital gains. Then it is necessary that the seller charge the buyer at least 9 percent simple interest or the IRS will imput interest at 10 percent compounded semi-annually. The required rate for land sales to family members is only 6 percent simple interest. By charging the lowest possible interest rate the property may sell for a greater amount.

Simple interest may also be attractive in family transactions even if the property is sold for a greater amount because of the lower interest charges. Lower interest and a higher sales price converts ordinary income to capital gain. With the resultant lower taxes, the family seller could lower the total cash payment (principal and interest) and receive the same amount after-taxes.

Combining graduated payments with simple interest is especially attractive since each simple interest payment, regardless how small, will consist of the same proportion of principal. Thus there will never be an increase in the unpaid balance of the loan. Also, since graduated payment schedules have their greatest appeal, and perhaps their only appeal, to sales between family members, it would seem appropriate to combine simple interest with graduated payments.

Simple Interest Payments

Debt is amortized under compound interest with level payments by the formula:

$$P = \frac{A}{(1 + i)} + \frac{A}{(1 + i)^2} + \ldots + \frac{A}{(1 + i)^n}$$

where P is the amount of debt, A is the level periodic payment consisting of principal and interest, i is the interest rate charged per period, and n is the number of periods. Each level payment, A, is broken up into an interest and principal component by first computing the interest that has accrued on the unpaid balance of the loan since the last payment, and since that amount is less than the payment A, the difference is principal reduction.

Debt is amortized under simple interest with level payments by the formula:

$$P = \frac{B}{(1 + i)} + \frac{B}{(1 + 2i)} + \ldots + \frac{B}{(1 + ni)}$$

where P, i and n are defined as before and B is the level periodic payment consisting of principal D and interest R, or B = D + R.

Simple interest R is computed as:

$$R = D\ (i\ s)$$

where i is the interest rate charged per period and s is the number of periods since the loan began. Substituting for R in the equation B = D + R results in:

$$B = D + D\ (i\ s).$$

Solving for D, the principal component, results in: D = B/(1 + i s) and then R, the interest component, is calculated as:

$$R = B - D.$$

With simple interest each payment is smaller than under compound interest, meaning that less interest is paid over the life of the payment schedule. This is because initial simple interest payments include more principal than do compound interest payments.

Graduated payments with compound interest have typically been geometric increases, where each payment increases by a set percentage through the life of the payment schedule, or for a stated number of periods and then remains constant. Geometric simple interest payments through the life of the payment schedule can be computed from the formula:

$$P = \frac{B}{(1 + i)} + \frac{B\,(1 + g)}{(1 + 2\,i)} + \ldots + \frac{B\,(1 + g)^{n-1}}{(1 + n\,i)}$$

where g is the geometric increase in payments. Simple interest on each payment would be computed as previously shown.

If it is desireable to only increase payments geometrically until the m th year, the formula is:

$$P = \frac{B}{(1 + i)} + \frac{B\,(1 + g)}{(1 + 2\,i)} + \ldots + \frac{B\,(1 + g)^{m-1}}{(1 + m\,i)} + \ldots + \frac{B\,(1 + g)^{m-1}}{(1 + n\,i)}$$

Although geometric payment increases have been most common, it is possible to compute other types of increases, such as an arithmetic increase, as:

$$P = \frac{B}{(1 + i)} + \frac{B\,(1 + g)}{(1 + 2\,i)} + \frac{B\,(1 + 2\,g)}{(1 + 3\,i)} + \ldots + \frac{B\,(1 + (n-1)\,g)}{(1 + n\,i)}$$

Obviously an enormous amount of flexibility exists to construct graduated payment schedules under both compound and simple interest. The advantage with simple interest is that each payment, regardless how small, will consist of some reduction in principle. Thus, unlike compound interest, there will never be an increase in the unpaid balance of the loan.

Examples of Geometric Increases

The author has written a computer program to generate simple interest, geometrically increasing payment plans (Appendix A). For comparison purposes a level simple interest payment schedule was generated for a $1,000 loan at 9 percent simple interest for 20 years (Table 2). The annual payment would be $89.88. In contrast, the compound interest level annual payment would be $109.55. A graduated compound interest payment plan would have to have a first payment of at least $90 or there would be an increase in the unpaid balance. A level simple interest payment is already a few cents less than that.

Table 3 is a simple interest payment schedule in which payments increase 2 percent a year. The first payment is $76.43, somewhat less than the level payment of $89.88. Because the first payment is lower, less principal is paid initially, so the final payment of $111.34 is larger than the level payment of $89.88. Notice, however, that the proportion of the first payment to principal reduction is .917 in both Tables 2 and 3. Total payments for the 2 percent geometrically increasing schedule are $1,856.97, only $59.27 more than the level payment schedule.

Table 2. Annual Simple Interest Debt Payment Schedule
$1,000 Debt, 9 Percent Simple Interest, 20-Year Term, Selected Years

Year	Payment	Interest	Principal	Balance
1	$ 89.88	$ 7.42	$ 82.46	$917.54
2	89.88	13.71	76.17	841.36
5	89.88	27.90	61.99	642.51
6	89.88	31.52	58.37	584.14
10	89.88	42.58	47.31	379.77
15	89.88	51.64	38.25	171.94
20	89.88	57.78	32.10	0.0
Totals	$1,797.70	$797.70	$1,000.00	

Table 3. Annual Simple Interest Geometric Graduated Debt Payment Schedule
$1,000 Debt, 9 Percent Simple Interest, 20-Year Term, Selected Years
Geometric Increases of 2 Percent

Year	Payment	Interest	Principal	Balance
1	$ 76.43	$ 6.31	$ 70.12	$929.88
2	77.96	11.89	66.06	863.82
5	82.73	25.67	57.05	684.52
6	84.38	29.59	54.79	629.73
10	91.34	43.26	48.07	428.34
15	100.84	57.93	42.91	204.51
20	111.34	71.58	39.76	0.0
Totals	$1,856.97	$856.97	$1,000.00	

In Table 4 the simple interest payments increase 6 percent a year and begin at a still lower amount of $53.70. Returning to the dairy example at the start of this paper, an investment of $5,500 per cow would then require a first payment of only $295, much less than the $440 available, leaving cash for living needs and capital replacement. If income increases average 5 percent a year over the next 20 years, as has been true over the last 20 years, then the increased annual payments should be relatively easy to meet. If not, then adjustments can be made. For instance, with 6 percent per year payment increases, by the 10th year, the payment on a $1,000 loan with 9 percent simple interest would be $90.72, still less than a 20 year level compound interest loan. And, the simple interest loan would have only $525 of the original loan to pay off, while the compound loan would still have about $710 left to pay. At the very least the simple interest loan could be converted to a level loan, either compound or simple, and still have lower payments than a continuing original conventional loan.

Table 4. Annual Simple Interest Geometric Graduated Debt Payment Schedule $1,000 Debt, 9 Percent Simple Interest, 20-Year Term, Selected Years Geometric Increases of 6 Percent

Year	Payment	Interest	Principal	Balance
1	$ 53.70	$ 4.43	$ 49.26	$950.74
2	56.92	8.68	48.23	902.50
5	67.79	21.04	46.75	761.22
6	71.86	25.20	46.66	714.56
10	90.72	42.97	47.75	525.87
15	121.40	69.74	51.66	276.42
20	162.46	104.44	58.02	0.0
Totals	$1,975.20	$975.20	$1,000.00	

Table 5 shows what happens with a graduated payment, 9 percent compound interest loan for 20 years. In order to start the payment at $53.70, it is necessary to increase each payment by 9.16 percent, a much faster rate of increase than the 6 percent increase under simple interest. The unpaid balance of the loan also increases beyond $1,000 until the 15th year because unpaid interest is accruing. Unlike the simple interest loan where the tenth payment is $90.72 and the unpaid balance is $525.87, the tenth payment is $118.29 and the unpaid balance is $1,193.20.

Table 5. Annual Compound Interest Geometric Graduated Debt Payment Schedule
$1,000 Debt, 9 Percent Compound Interest, 20-Year Term, Selected Years
First Payment $53.70

Year	Payment	Interest	Principal	Balance
1	$ 53.70	$ 53.70	$ 0.0	$1,036.30
2	58.62	58.62	0.0	1,070,95
5	76.25	76.25	0.0	1,158.50
6	83.24	83.24	0.0	1,179.52
10	118.21	118.21	0.0	1,193.20
15	183.25	105.62	77.63	922.37
20	287.15	23.71	263.44	0.0
Totals	$2,801.19	$1,801.19	$1,000.00	

Example of Arithmetic Increases

The concept of simple interest involves arithmetic increases in the discount rate, while compound interest involves geometric increases. Intuitively then, it might be appropriate to schedule arithmetic increases in simple interest payments rather than geometric increase. Although income might be expected to increase geometrically, arithmetic increases in payments would not appear as ominous to borrowers as geometric payments.

Table 6 is a 20 year, 9 percent simple interest payment schedule for a $1,000 loan with payments increasing $5.00 per year. The first payment of $50.57 is lower than the $53.70 simple interest first payment with geometric increases of 6 percent. The final payment for the 20th year is only $145.57 with arithmetic rather than $162.46 with the geometric increases. The total payments of the arithmetic increase payment plan are only $1961.37, compared to $1975.20 with the geometric payment plan.

Table 6. Annual Simple Interest Arithmetic Graduated Debt Payment Schedule
$1,000 Debt, 9 Percent Simple Interest, 20-Year Term, Selected Years
Arithmetic Increases of $5.00

Year	Payment	Interest	Principal	Balance
1	$ 50.57	$ 4.18	$ 46.39	$ 953.61
2	55.57	8.48	47.09	906.52
5	70.57	21.90	48.67	761.94
6	75.57	26.50	49.07	712.87
10	95.57	45.27	50.30	513.36
15	120.57	69.26	51.31	258.67
20	145.57	93.58	51.99	0.0
Totals	$1,961.37	$ 961.37	$1,000.00	

Conclusions

Although there have been numerous discussions of graduated payment mortgages to finance the transfer of farms, there appears little incentive for commercial lenders to sponsor them. To be effective under compound interest, the initial payments must be less than the accrued interest, resulting in an increase in the loan balance. This gives an incentive to default if the value of the collateral does not also increase. The increase in the unpaid balance also requires enormous payments towards the end of the payment plan, while incomes may not increase sufficiently to support these higher payments.

It appears that graduated payment plans may only be feasible in family transactions. In family transactions it is already attractive to use simple interest rather than compound interest in scheduling loan payments because it is the IRS's lowest required interest charge. Graduated payments are even more attractive with simple interest than with compound interest, because with simple interest there is never an increase in the unpaid balance, regardless how small the first payment. With a graduated payment simple interest payment schedule, more interest will be paid than with level simple interest, but the increase is not as great as the difference under graduated and non-graduated compound interest. Thus, the percentage increase in payments with simple interest does not have to be as great as with compound interest, making graduated simple interest payment schedules feasible.

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APPENDIX A: SIMPLE INTEREST GEOMETRIC GRADUATED LOAN PROGRAM

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00002 REM SIMPLE INTEREST GEOMETRIC GRADUATED PROGRAM
00003 REM WRITTEN BY LOREN TAUER, DEPT. OF AG. ECON., CORNELL UNIV.
00005 PRINT 'ENTER LOAN AMOUNT'
00010 INPUT P
00015 PRINT ' ENTER INTEREST RATE PER PERIOD'
00020 INPUT R
00025 PRINT ' ENTER NUMBER OF LOAN PAYMENTS'
00030 INPUT N
00040 PRINT ' ENTER GEOMETRIC INCREASE'
00045 INPUT G
00046 REM L AND U ARE LOWER AND UPPER BOUNDS FOR A.
00047 REM THEY WILL BE INCREASE AND DECREASED UNTIL THE SOLUTION FOR
00048 REM  A IS FOUND.
00049 REM A IS THE INITIAL PAYMENT WHICH INCREASES G PERCENT EACH PAYMENT.
00050 L=0.
00060 U=P*10.
00070 A=U
00080 S=0.
00081 SAA=0.
00082 ST=0.
00083 SV=0.
00090 FOR I=1 TO N
00100 S=S+(A*(1.+G)**(I-1)/(1+I*R))
00110 NEXT I
00120 C=S-P
00130 IF C>.001 THEN 180
00140 IF C>-.001 THEN 210
00141 REM SET THE LOWER BOUND EQUAL TO A AND TRY HALFWAY BETWEEN THIS
00142 REM  NEW L AND U.
00150 L=A
00160 A=(U+L)/2
00170 GO TO 80
00171 REM SET THE UPPER BUUNC EQUAL TO A AND TRY HALFWAY BETWEEN THIS
00172 REM  NEW U AND L.
00180 U=A
00190 A=(U+L)/2
00200 GO TC 80
00210 PRINT 'SIMPLE INTEREST PAYMENTS'
00211 PRINT 'PAYMENT','AMOUNT','PRINCIPAL','INTEREST','BALANCE'
00212 REM COMPUTE COMPONENTS OF EACH PAYMENT AND PRINT
00220 FOR J=1 TO N
00230 AA=A*(1.+G)**(J-1)
00231 T=AA/(1+J*R)
00232 V=AA-T
00233 P=P-T
00250 PRINT J,AA,T,V,P
00251 SAA=SAA+AA
00252 ST=ST+T
00253 SV=SV+V
00260 NEXT J
00261 PRINT 'TOTALS'
00262 PRINT ,SAA,ST,SV
00270 END
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